An Economic Model of “Fulfilled By Amazon” (FBA)

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Introduction

- Fulfillment by Amazon (FBA): Amazon helps merchants do delivery for a per unit fee

- Question: what are the welfare implications of FBA upon consumers, merchants and Amazon?

- Answer: 2 outcomes:
  1. Consumers, merchants with low fulfillment service quality and Amazon gain, but merchants with high fulfillment service quality lose
  2. Amazon extracts all gains of FBA, others stay the same

Model

- Static setting: time $t \in \{0, 1, 2\}$
- 3 types of agents: consumers, 2 merchants and a platform

- $\underline{t = 0}$: platform sets platform fee based on revenue and FBA fee per quantity
- $\underline{t = 1}$: each merchant decides if to use platform/ FBA and sets price
- $\underline{t = 2}$: consumers buy from one merchant or an outside option

Merchants

- Two merchants defined by
  - $\theta_j > 0$: product quality
  - $\sigma_j \geq 0$: fulfillment service quality

- Indexed by H, L : $\sigma_H > \sigma_L$, we allow $\theta_H \leq \theta_L$, normalize $\sigma_L \equiv 0$

- At $t = 1$ merchant $j \in \{H, L\}$ simultaneously makes 3 decisions:
  1. $\rho_j \in \{0, 1\}$: join platform with fee f or not
  2. $\eta_j \in \{0, 1\}$: use FBA or with fee T not, using FBA changes σ_j to $\sigma_P \equiv \sigma_H$ $(\rho_j = 0 \Rightarrow \eta_j = 0)$; FBA useless to H
  3. P_j: price

Merchants' problem

Merchants solve

$$d_j^\star(f,T) = (\rho_j^\star(f,T), \eta_j^\star(f,T), P_j^\star(f,T)) \in \underset{\rho_j\in\{0,1\},\eta_j\in\{0,1\},P_j\geq 0}{\arg\max} \Pi_j(\rho_j, \eta_j, P_j, d_{-j})$$

$\Pi_j(\rho_j, \eta_j, P_j, d_{-j})$: profit of Merchant j

$Q_j((\rho_j, \eta_j, P_j), d_{-j})$: demand of Merchant j

f : platform fee

T : FBA fee
Assume cost of production and for FBA is zero

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$$\Pi_j(\rho_j,\eta_j,P_j,d_{-j}) = \begin{cases} P_j \cdot Q_j & \text{if } \rho_j = 0 \\ P_j \cdot Q_j - f \cdot P_j \cdot Q_j & \text{if } \rho_j = 1, \eta_j = 0 \\ P_j \cdot Q_j - f \cdot P_j \cdot Q_j - T \cdot Q_j & \text{if } \rho_j = 1, \eta_j = 1 \end{cases}$$

Platform's problem

Platform anticipates $P_j^\star(f,T), Q_j^\star(f,T)$ given fees (f,T), and sets fees optimally to solve:

$$(f^\star, T^\star) \in \underset{f \in [0,1], T \geq 0}{\arg\max}\ \Pi_P(f,T;d)$$

$$\Pi_P(f,T;d) = f \cdot \left(\sum_{j \in M_P^\star(f,T)} \underbrace{P_j^\star(f,T) \cdot Q_j^\star(d(f,T))}_{\text{revenue of } j} \right) + T \cdot \left(\sum_{j \in M_F^\star(f,T)} \underbrace{Q_j^\star(d(f,T))}_{\text{demand of } j} \right)$$

$M_P^\star(f,T) =$ set of merchants using platform

$M_F^\star(f,T) =$ set of merchants using FBA $\subseteq M_P^\star(f,T)$

Consumers

- A measure of consumers indexed by $i \in [0, 1 + \Delta]$
- Utility $U_i(j, d_j)$ of consumer i from buying product j as a function of Merchant j's decisions, $d_j = (\rho_j, \eta_j, P_j)$:

$$U_i(j, d_j) = \underbrace{X_{i,j}}_{\text{Consumption Value}} - \underbrace{P_j}_{\text{product Price}} + \underbrace{s_j(\rho_j, \eta_j)}_{\text{Fulfillment Service Value}}$$

- $X_{i,j} \sim \exp(\theta_j^{-1})$, independent across i, j, $\mathbb{E}(X_{i,j}) = \theta_j$

$$s_j(\rho_j, \eta_j) = \begin{cases} \sigma_P \equiv \sigma_H & \text{if } \rho_j \cdot \eta_j = 1 \\ \sigma_j & \text{otherwise} \end{cases}$$

- Outside option with utility 0

Demand function

- Consumer $i \in [0,1]$ sees product sold by all merchants
- Consumer $i \in (1, 1+\Delta]$ sees product j only if Merchant j sells on the platform, i.e., $\rho_j = 1$

Demand of Merchant j after both merchants make decisions $d = (d_L, d_H)$

$$Q_{j,[0,1]}(d_j, d_{-j}) = \mathbb{P}\Big(U_i(j, d_j) = \max\{U_i(H, d_H), U_i(L, d_L), 0\}, i \in [0,1]\Big)$$

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$$Q_{j,\Delta}(d_j, d_{-j}) = \rho_j \Delta\, \mathbb{P}\Big(U_i(j, d_j) = \max_{k \in M_P^\star(f,T)} \{U_i(k, d_k))\} \vee 0, i \in (1, 1+\Delta]\Big)$$

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- Platform: strictly better:
  1. When effects on merchants and consumers are zero: platform takes all values generated through FBA
  2. Otherwise: only H loses, everyone else gains

Main results (effects of FBA) – 2

When will *H* lose; *L*, consumers, and platform gain?

- θ_L is small
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When will H lose; L, consumers, and platform gain?

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1. θ_L small: Q_L more sensitive to $T \uparrow$, more difficult to extract values from FBA

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When will H lose; L, consumers, and platform gain?

- θ_L is small
- $T^\star$ is "interior": L strictly prefers to use FBA

1. θ_L small: Q_L more sensitive to $T\uparrow$, more difficult to extract values from FBA
2. θ_L large: Q_L less sensitive to $T\uparrow$, easier to extract values from FBA

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- Merchant H, even though not using FBA, weakly loses
- Amazon can extract all values from FBA if θ_L is large
- Future directions: extend the model to more than 2 merchants

Thank you! Questions?

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Full paper link: